	Consider the curve defined by the equation $2y^3 + 6x^2y - 12x^2 + 6y = 1$ with $\dfrac{dy}{dx} = \dfrac{4x - 2xy}{x^2 + y^2 + 1}$ b) Write an equation of each horizontal tangent to the curve c) The line through the origin with slope -1 is tangent to the curve at point P. Find the x and y-coordinates of P. d) Find $\dfrac{d^2y}{dx^2}$ in terms of x and y.

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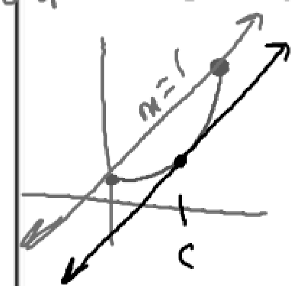
MVT

AROC = IROC

Secant slope = Tangent slope

slope = derivative



2013 BC3

Hot water is dripping through a coffeemaker, filling a large ~~cup~~ with coffee. The amount of coffee in the cup at time t, $0 \le t \le 6$, is given by a differentiable function C, where t is measured in minutes. Selected values of C(t), ~~measured in ounces~~, are given in the table.

t(minutes)	0	1	2	3	4	5	6
C(t) ounces	0	5.3	8.8	11.2	12.8	13.8	14.2

Is there a time t, $3 \le t \le 6$, at which $C'(t) = 1$. Justify your answer.

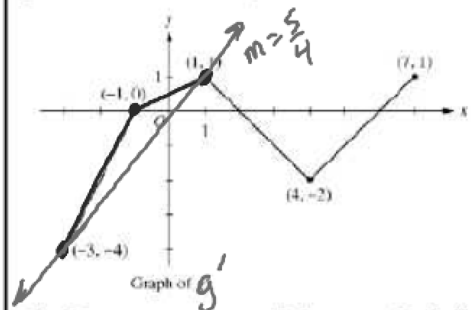
(3, 11.2) (6, 14.2)

$C'(t) = 1$

$AROC = \frac{14.2 - 11.2}{6 - 3} = 1$

Yes, because C(t) is differentinable from (3, 6) and continuous [3, 6] and the AROC = C′(t)

Let g be a continuous function with g(2) = 5. The graph of the piecewise-linear function g', the derivative of g, is shown for $-3 \le x \le 7$.



Find the average rate of change of $g'(x)$, on the interval $-3 \le x \le 1$. Does the Mean Value Theorem applied on the interval $-3 \le x \le 1$ guarantee a value of c, for -3 < c < 1, such that $g''(c)$ is equal to this average rate of change? Why or why not?

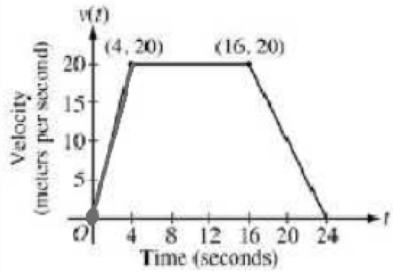
(−3, −4) (1, 1)

$AROC = \frac{-4-1}{-3-1} = \frac{-5}{-4} = \frac{5}{4}$

No, because g′ is not differentiable at x = −1

2005 AB5

A car is traveling on a straight road. For $0 \le t \le 24$ seconds, the car's velocity v(t), in meters per second, is modeled by the piecewise-linear function defined by the graph



Find the average rate of change of v over the interval $0 \le t \le 16$. Does the Mean Value guarantee a value of c, for 0 < c < 16, such that $v'(t)$ is equal to this average rate of change? Why of why not?

2004 BCB3

A test plane flies in a straight line with positive velocity v(t), in miles per minute at time t minutes, where v is a differentiable function of t. Selected values of v(t) are shown.

t(min)	0	5	10	15	20	25	30	35	40
v(t) (mpm)	7	9.2	9.5	9.2	4.5	2.4	4.5	4.9	7.3

(5,15)

$AROC = \frac{9.2-9.2}{15-5} = \frac{0}{10}$

$AROC = \frac{4.5-4.5}{30-20} = 0$

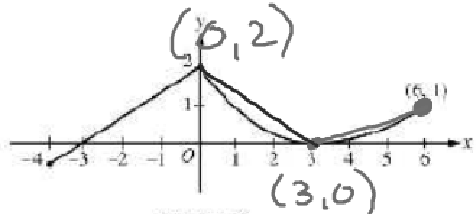
(20,30

Based on the values in the table, what is the smallest number of instances at which the acceleration of the plane could equal zero on the open interval 0 < t < 40? Justify your answer

Two. Since v(t) is differentiable from (0,40) and continuous from [0,40] the AROC = a(t) from (5,15) and (20,30)

2009 BC3

A continuous function f is defined on the closed interval $-4 \le x \le 6$. The graph of f consists of a line segment and a curve that is tangent to the x-axis at x = 3, as shown in the figure above. On the interval 0 < x < 6, the function f is twice differentiable, with $f''(x) > 0$.



Graph of f

Is there a value a, for which the Mean Value Theorem, applied to the interval [a, 6], guarantees a value c, a < c < 6, at which $f'(c) = \frac{-1}{6}$? Justify your answer.

(a, 2)

(a, 0) (6, 1)

a = 0

Since differentiable (0, 6)

continuous [0, 6)

AROC

$f'(c) = -\frac{1}{6}$

2011 BCB5

Ben rides a unicycle back and forth along a straight east-west track. The twice-differentiable function B models Ben's position of the track, measured in meters from the western end of the track, at time t, measured in seconds from the start of the ride. The table gives values of B(t) and Ben's velocity, v(t), measured in meters per second, at selected times t.

t(seconds)	0	15	40	60
B(t) (meters)	100	136	9	46
V(t) meters per second	2	2.3	2.5	4.6

For $15 \le t \le 60$, must there be a time t when Ben's velocity is -2 meters per second? Justify your answer.

4) $\lim_{x\to 1} \frac{\sqrt[3]{x}-1}{x-1}$

49) $\lim_{x\to 1} \frac{x^3-1}{4x^3-x-3}$

$\frac{\infty}{\infty}$ ← A) $\lim_{x\to\infty} \frac{x^3-1}{4x^3-x-3} = \frac{1}{4}$

$\frac{\infty}{\infty}$ ← $\lim_{x\to\infty} \frac{3x^2}{12x^2-1}$

$\lim_{x\to\infty} \frac{6x}{24x} = \lim_{x\to\infty} \frac{6}{24}$

27) $\boxed{\lim_{x\to\infty} \frac{\ln(x^5)}{x}} = \frac{\infty}{\infty}$

$\lim_{x\to\infty} \frac{\frac{1}{(x^5)}\cdot 5x^4}{1}$

$\lim_{x\to\infty} \frac{5}{x} = 0$

35) $\lim_{x\to\infty} \frac{\boxed{\log_2(x)}}{\boxed{\log_3(x+3)}} = \frac{\infty}{\infty}$

$\lim_{x\to\infty} \frac{\left(\frac{1}{x\ln 2}\right)}{\left(\frac{1}{(x+3)\ln 3}\right)}$

$\boxed{\lim_{x\to\infty} \frac{(x+3)\ln 3}{x\ln 2} =}$

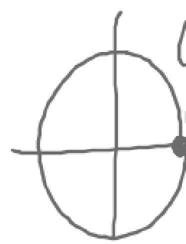
$\lim_{x\to\infty} \frac{\ln 3}{\ln 2} = \frac{\ln 3}{\ln 2}$

$\frac{\left(\frac{1}{5}\right)}{\left(\frac{1}{3}\right)}$

$\frac{1}{5} \div \frac{1}{3}$

$\frac{1}{5} \cdot \frac{3}{1}$

33) $\lim_{x\to 0} \frac{\sin(x^2)}{x} = \frac{0}{0}$



$\lim_{x\to 0} \frac{2x\cos(x^2)}{1} = \frac{0}{1} = 0$

$2(0)\cos(0)$

angle